Semi-Annual Status Report

NASA Research Grant NsG-540/22-05-002

Brandeis University

Period October 1, 1964 to March 31, 1965

During the report period, research has been carried out in four major problems. In the cases of Messrs. Gilbert and Burke, the work they have done in the Brandeis Astrophysics Institute has developed out of problems begun by them at Harvard University, in the Departments of Physics and Astronomy, respectively. Their separate researches, the major portions of which were supported by this grant, have resulted in doctoral theses which have been submitted to and accepted by Harvard University. The titles are as follows:

1. "The Statistics of Gravitational Clustering" by Ira Gilbert.
2. "Some Star-Star and Star-Gas Interactions" by Anthony Burke.

An abstract of Dr. Gilbert's work is included as an appendix to this report. Dr. Burke's paper is in the process of being reproduced and a copy will be forwarded as soon as it is available, which should be within the next few weeks. In the meantime, the table of contents of Dr. Burke's paper is included in this report as Appendix II.

An effort to obtain a suitable statistical description of a large stellar association such as a globular cluster has been carried out by Mr. Paul Easton under the supervision of Dr. J. Goldstein. This work will be presented to the faculty of Brandeis University as a doctoral thesis in the latter part of June 1965. The problem has been to find a description in the E-J (energy-angular momentum) space, rather than in the conventional position- velocity space. It has been possible to derive a Fokker-Planck equation for the distribution function, although the usual problems associated with the long-range-unscreened Coulomb forces still arise, and artificial cutoffs must be introduced. The cutoff is of the order of magnitude of the cluster size. The results are relatively insensitive to the choice of cutoff parameter, since this parameter only appears in a logarithm. The integrals which arise are presently being evaluated on the IBM 7040 at the MIT Computation Center, and it is anticipated that the results will be in hand shortly.

A problem in radiative transfer has been investigated by J. Goldstein. Here we are concerned with an atmosphere which is only approximately plane parallel; a perturbation-type calculation is then possible. We assume that the local density can be written $\rho = \rho_0 + \rho_1$ with $\rho_1 \ll \rho_0$; ρ_0 depends on the vertical coordinate, z, only and ρ_1 has the form $\rho_1 = \rho_1(z)\, e^{ipx} e^{iqy}$. The local temperature is assumed to have the same structure. The radiation field can then be written as $I_0 + I_1$, where, for the case of a grey atmosphere with isotropic scattering, I_0 is the solution of the standard Milne problem. We show that the problem reduces to another Milne-type equation, but in two dimensions:

$$\Phi(\tau,\mu) = \frac{\omega}{2}\int_0^1 \frac{d\mu'}{\mu'}\int_0^\infty d\tau'\, e^{-|\tau-\tau'|/\mu'}\,\Phi(\tau',\mu')\, J_0\left\{\frac{\sqrt{1-\mu'^2}}{\mu'}\nu r\right.$$

$$+\ \alpha B_1(\tau) + \frac{\rho_1}{\rho_0}\mu\frac{\partial I_0(\tau,\mu)}{\partial\tau}$$

where, if Φ can be solved for, we obtain I_1, from

$$I_1 = -\int_\tau^\infty d\tau'\, e^{-(\tau-\tau')/\mu}\frac{1}{\mu}\Phi(\tau',\mu)\, e^{-i\int_{\tau'}^{\tau}\frac{d\tau''}{\mu}\frac{\sqrt{1-\mu^2}}{\rho_0 k}[p\cos\varphi + q\sin\varphi]} \qquad (\mu>0)$$

with a similar expression holding for $\mu<0$.

The notation is explained as follows (for the most part, the notation is the same as in Chandrasekhar's monograph):

ω = local albedo (assumed constant)

μ = $\cos\theta$ (angle with z-axis)

$\cos\varphi = \frac{x}{\sqrt{x^2+y^2}}$

τ = optical depth, defined from the zero-order Milne problem

$B(\tau) = B_0(\tau) + B_1(\tau)\, e^{ipx} e^{iqy}$

= local black body function

$\alpha = 1-\omega$

We observe that even in the absence of temperature fluctuations, the first order radiation intensity does not vanish, but that the zero order solution acts as a source term for the first order intensity. Thus, the straightforward interpretation of solar granulations, for example, as arising solely from temperature differences is called into question.

We are at present seeking ways of extracting numerical information from this rather formidable integral equation.

Finally, we mention that a short paper by Dr. Gilbert, entitled "The Development of Irregularities in an Expanding Universe" has been prepared and will shortly be submitted to the Astrophysical Journal for publication.

Appendix II

SOME STAR-STAR AND STAR-GAS INTERACTIONS*

by

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*Submitted to Harvard University in partial fulfillment of the requirements for the Ph.D.

*Supported in part by NASA research grant NSG 540-22/05-002.

THE STATISTICS OF GRAVITATIONAL CLUSTERING*

by Ira Gilbert
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ABSTRACT

The research described in this thesis consists of an application of the hierarchical distribution function formalism to the cosmogonical problem of the development of irregularities in an expanding universe.

The cosmological setting of the work to follow is first presented in a brief resume of certain aspects of Layzer's hypothesis of gravitational clustering. Next, the equations of motion of a cosmic distribution of mutually gravitating point masses are obtained in a particularly convenient set of position, velocity and time coordinates. In these coordinates the gross expansion of the universe is transformed away and in its place an apparent background of negative mass and an explicit time dependence of the gravitational constant appear.

The formalism of distribution and correlation functions is then developed with emphasis placed upon the generating functional of Bogolioubov. The generating functional method is employed in a new derivation of the equations of motion for the correlation functions. Layzer's clustering spectrum is redefined in terms of the two-particle correlation function to

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obtain a quantity more appropriate to a particulate distribution.

Various models of clustering are next studied and the simplicity of the correlation functions as opposed to the distribution functions exhibited. The clustering spectrum for a three-level hierarchical distribution of mass points is explicitly calculated with the aid of the generating functional. It is found that pronounced clustering on a distance scale λ does not necessarily imply a peak in the clustering spectrum at $k \approx 1/\lambda$ as one might intuitively expect.

The dynamics of clustering are then investigated and the energy theorem for the peculiar kinetic and potential energies derived. The simplification resulting from the assumption of weak clustering is exploited in the derivation of a useful integral equation for the clustering spectrum. This integral equation is numerically solved under the assumption of certain simple initial conditions. As expected, it is found that clustering on a distance scale much smaller than Jeans' critical wavelength is overcome by Landau damping or phase mixing but that clustering on a much larger scale proceeds to grow.